

On identities in Hom-Malcev algebras

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Abstract

In a Hom-algebra an identity, equivalent to the Hom-Malcev identity, is found.

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1 Introduction and statement of results

Hom-Lie algebras were introduced in [3] as a tool in understanding the structure of some q-deformations of the Witt and the Virasoro algebras. Since then, the theory of Hom-type algebras began an intensive development (see, e.g., [2], [4], [6], [7], [8], [12], [13], [14], [15]). Hom-type algebras are defined by twisting the defining identities of some well-known algebras by a linear self-map, and when this twisting map is the identity map, one recovers the original type of considered algebras.

In this setting, a Hom-type generalization of Malcev algebras (called Hom-Malcev algebras) is defined by D. Yau in [15]. Recall that a *Malcev algebra* is a nonassociative algebra (A, \cdot) , where the binary operation “ \cdot ” is anti-commutative, such that the identity

$$J(x, y, x \cdot z) = J(x, y, z) \cdot x \quad (1.1)$$

holds for all $x, y, z \in A$ (here $J(x, y, z)$ denotes the Jacobian, i.e. $J(x, y, z) = xy \cdot z + yz \cdot x + zx \cdot y$). The identity (1.1) is known as the *Malcev identity*. Malcev algebras were introduced by A.I. Mal'tsev [9] (calling them Moufang-Lie algebras) as tangent algebras to local smooth loops, generalizing in this way a result in Lie theory stating that a Lie algebra is a tangent algebra to a local Lie group (in fact, Lie algebras are special case of Malcev algebras). Another approach to Malcev algebras is the one from alternative algebras:

every alternative algebra is Malcev-admissible [9]. So one could say that the algebraic theory of Malcev algebras started from Malcev-admissibility of algebras. The foundations of the algebraic theory of Malcev algebras go back to E. Kleinfeld [5], A.A. Sagle [10] and, as mentioned in [10], to A.A. Albert and L.J. Paige. Some twisting of the Malcev identity (1.1) along any algebra self-map α of A gives rise to the notion of a Hom-Malcev algebra (A, \cdot, α) ([15]; see definitions in section 2). Properties and constructions of Hom-Malcev algebras, as well as the relationships between these Hom-algebras and Hom-alternative or Hom-Jordan algebras are investigated in [15]. In particular, it is shown that a Malcev algebra can be twisted into a Hom-Malcev algebra and that Hom-alternative algebras are Hom-Malcev admissible.

In [15], as for Malcev algebras (see [10], [11]), equivalent defining identities of a Hom-Malcev algebra are given. In this note, we mention another identity in a Hom-Malcev algebra that is equivalent to the ones found in [15]. Specifically, we shall prove the following

Theorem. *Let (A, \cdot, α) be a Hom-Malcev algebra. Then the identity*

$$J_\alpha(w \cdot x, \alpha(y), \alpha(z)) = J_\alpha(w, y, z) \cdot \alpha^2(x) + \alpha^2(w) \cdot J_\alpha(x, y, z) - 2J_\alpha(y \cdot z, \alpha(w), \alpha(x)) \quad (1.2)$$

holds for all w, x, y, z in A , where $J_\alpha(x, y, z) = xy \cdot \alpha(z) + yz \cdot \alpha(x) + zx \cdot \alpha(y)$. Moreover, in any anti-commutative Hom-algebra (A, \cdot, α) , the identity (1.2) is equivalent to the Hom-Malcev identity

$$J_\alpha(\alpha(x), \alpha(y), x \cdot z) = J_\alpha(x, y, z) \cdot \alpha^2(x) \quad (1.3)$$

for all x, y, z in A .

Observe that when $\alpha = Id$ (the identity map) in (1.3), then (1.3) is (1.1) i.e. the Hom-Malcev algebra (A, \cdot, α) reduces to the Malcev algebra (A, \cdot) (see [15]).

In section 2 some instrumental lemmas are proved. Some results in these lemmas are a kind of the Hom-version of similar results by E. Kleinfeld [5] in case of Malcev algebras. The section 3 is devoted to the proof of the theorem.

Throughout this note we work over a ground field \mathbb{K} of characteristic 0.

2 Definitions. Preliminary results

In this section we recall useful notions on Hom-algebras ([8], [12], [13], [15]), as well as the one of a Hom-Malcev algebra [15]. In [5], using an analogue of the Bruck-Kleinfeld function, an identity (see identity (6) in [5]) characterizing Malcev algebras is found. This identity is used in [10] to derive further identities for Malcev algebras (see [10], Proposition 2.23). The main result of this section (Lemma 2.7) proves that the Hom-version of the identity (6) of [5] holds in any Hom-Malcev algebra.

Definition 2.1. A *multiplicative Hom-algebra* is a triple (A, μ, α) , in which A is a \mathbb{K} -module, $\mu : A \times A \rightarrow A$ is a bilinear map (the binary operation), and $\alpha : A \rightarrow A$ is a linear map (the twisting map) such that α is an endomorphism of (A, μ) . The Hom-algebra (A, μ, α) is said *anticommutative* if the operation μ is skew-symmetric, i.e. $\mu(x, y) = -\mu(y, x)$, for all $x, y \in A$.

In the rest of this paper, we will use the abbreviation $x \cdot y = \mu(x, y)$ in a Hom-algebra (A, μ, α) .

Remark. The multiplicativity of the twisting map is not necessary in the definition of a Hom-algebra (see, e.g., [6], [8]). The multiplicativity is included here for convenience.

Definition 2.2. Let (A, \cdot, α) be an anticommutative Hom-algebra.

- (i) The *Hom-Jacobian* ([8]) of (A, \cdot, α) is the trilinear map $J_\alpha(x, y, z)$ on A defined by $J_\alpha(x, y, z) = xy \cdot \alpha(z) + yz \cdot \alpha(x) + zx \cdot \alpha(y)$.
- (ii) (A, \cdot, α) is called a *Hom-Lie algebra* ([3]) if the *Hom-Jacobi identity* $J_\alpha(x, y, z) = 0$ holds in (A, \cdot, α) .

Definition 2.3. ([15]) A *Hom-Malcev algebra* is an anticommutative algebra (A, \cdot, α) such that the *Hom-Malcev identity* (see (1.3))

$$J_\alpha(\alpha(x), \alpha(y), x \cdot z) = J_\alpha(x, y, z) \cdot \alpha^2(x)$$

holds in (A, \cdot, α) .

Remark. When $\alpha = Id$, then the Hom-Jacobi identity reduces to the usual Jacobi identity $J(x, y, z) := xy \cdot z + yz \cdot x + zx \cdot y = 0$, i.e. the Hom-Lie algebra (A, \cdot, α) reduces to the Lie algebra (A, \cdot) . Likewise, when $\alpha = Id$, the Hom-Malcev identity reduces to the Malcev identity (1.1), i.e. the Hom-

Malcev algebra (A, \cdot, α) reduces to the Malcev algebra (A, \cdot) .

The following simple lemma holds in any anticommutative Hom-algebra.

Lemma 2.4. *In any anticommutative Hom-algebra (A, \cdot, α) the following holds:*

- (i) $J_\alpha(x, y, z)$ is skew-symmetric in its three variables.
- (ii) $\alpha^2(w) \cdot J_\alpha(x, y, z) - \alpha^2(x) \cdot J_\alpha(y, z, w) + \alpha^2(y) \cdot J_\alpha(z, w, x)$
 $- \alpha^2(z) \cdot J_\alpha(w, x, y)$
 $= J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x)) + J_\alpha(w \cdot y, \alpha(z), \alpha(x))$
 $+ J_\alpha(z \cdot x, \alpha(w), \alpha(y)) - J_\alpha(z \cdot w, \alpha(x), \alpha(y)) - J_\alpha(x \cdot y, \alpha(z), \alpha(w)),$

for all w, x, y, z in A .

Proof. The skew-symmetry of $J_\alpha(x, y, z)$ in w, x, y, z follows from the skew-symmetry of the operation “.”.

Expanding the expression in the left-hand side of (ii) and then rearranging terms, we get (by the skew-symmetry of “.”)

$$\begin{aligned}
& \alpha^2(w) \cdot J_\alpha(x, y, z) - \alpha^2(x) \cdot J_\alpha(y, z, w) + \alpha^2(y) \cdot J_\alpha(z, w, x) \\
& - \alpha^2(z) \cdot J_\alpha(w, x, y) \\
& = -\alpha^2(z) \cdot (wx \cdot \alpha(y)) + \alpha^2(y) \cdot (wx \cdot \alpha(z)) \\
& - \alpha^2(x) \cdot (yz \cdot \alpha(w)) + \alpha^2(w) \cdot (yz \cdot \alpha(x)) \\
& - \alpha^2(x) \cdot (wy \cdot \alpha(z)) - \alpha^2(z) \cdot (yw \cdot \alpha(x)) \\
& + \alpha^2(w) \cdot (zx \cdot \alpha(y)) + \alpha^2(y) \cdot (xz \cdot \alpha(w)) \\
& - \alpha^2(x) \cdot (zw \cdot \alpha(y)) + \alpha^2(y) \cdot (zw \cdot \alpha(x)) \\
& + \alpha^2(w) \cdot (xy \cdot \alpha(z)) - \alpha^2(z) \cdot (xy \cdot \alpha(w)).
\end{aligned}$$

Next, adding and subtracting $\alpha(yz) \cdot \alpha(wx)$ (resp. $\alpha(wx) \cdot \alpha(yz)$, $\alpha(zx) \cdot \alpha(wy)$, $\alpha(wy) \cdot \alpha(zx)$, $\alpha(xy) \cdot \alpha(zw)$ and $\alpha(zw) \cdot \alpha(xy)$) in the first (resp. second, third, fourth, fifth, and sixth) line of the right-hand side expression in the last equality above, we come to the equality (ii) of the lemma. \square

In a Hom-Malcev (A, \cdot, α) we define the multilinear map G by

$$\begin{aligned}
G(w, x, y, z) &= J_\alpha(w \cdot x, \alpha(y), \alpha(z)) - \alpha^2(x) \cdot J_\alpha(w, y, z) \\
&\quad - J_\alpha(x, y, z) \cdot \alpha^2(w)
\end{aligned} \tag{2.1}$$

for all w, x, y, z in A .

Remark. (1) If $\alpha = Id$ in (2.1), then $G(w, x, y, z)$ reduces to the function $f(w, x, y, z)$ defined in [5] which in turn is a variation of the Bruck-Kleinfeld function defined in [1].

(2) If in (2.1) replace $J_\alpha(t, u, v)$ with the *Hom-associator* [8] $as(t, u, v)$, then one recovers the Hom-Bruck-Kleinfeld function defined in [15].

Lemma 2.5. *In a Hom-Malcev algebra (A, \cdot, α) the function $G(w, x, y, z)$ defined by (2.1) is skew-symmetric in its four variables.*

Proof. From the skew-symmetry of “ \cdot ” and $J_\alpha(t, u, v)$ (see Lemma 2.4(i)) it clearly follows that

$$G(x, w, y, z) = -G(w, x, y, z)$$

$$G(w, x, z, y) = -G(w, x, y, z).$$

Next, using the skew-symmetry of $J_\alpha(t, u, v)$,

$$\begin{aligned} G(y, x, y, z) &= J_\alpha(y \cdot x, \alpha(y), \alpha(z)) - J_\alpha(x, y, z) \cdot \alpha^2(y) \\ &= J_\alpha(\alpha(y), \alpha(z), y \cdot x) - J_\alpha(y, z, x) \cdot \alpha^2(y) \\ &= J_\alpha(y, z, x) \cdot \alpha^2(y) - J_\alpha(y, z, x) \cdot \alpha^2(y) \quad (\text{by (1.3)}) \\ &= 0. \end{aligned}$$

Likewise, one checks that $G(w, y, y, z) = 0$. This suffices to prove the skew-symmetry of $G(w, x, y, z)$ in its variables. \square

As we shall see below, the following lemma is a consequence of the definition of $G(w, x, y, z)$ and the skew-symmetry of $J_\alpha(t, u, v)$ and $G(w, x, y, z)$.

Lemma 2.6. *Let (A, \cdot, α) be a Hom-Malcev . Then*

$$\begin{aligned} &J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(x \cdot y, \alpha(z), \alpha(w)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x)) \\ &+ J_\alpha(z \cdot w, \alpha(x), \alpha(y)) = 0; \end{aligned} \quad (2.2)$$

$$\begin{aligned} &2G(w, y, y, z) - \alpha^2(w) \cdot J_\alpha(x, y, z) + \alpha^2(x) \cdot J_\alpha(w, y, z) - \alpha^2(y) \cdot J_\alpha(z, w, x) \\ &+ \alpha^2(z) \cdot J_\alpha(w, x, y) = J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x)), \end{aligned} \quad (2.3)$$

for all w, x, y, z in A .

Proof. From the definition of $G(w, x, y, z)$ (see (2.1)) we have $J_\alpha(w \cdot x, \alpha(y), \alpha(z)) = G(w, x, y, z) + \alpha^2(x) \cdot J_\alpha(w, y, z) + J_\alpha(x, y, z) \cdot \alpha^2(w)$,

$$\begin{aligned}
J_\alpha(x \cdot y, \alpha(z), \alpha(w)) &= G(x, y, z, w) + \alpha^2(y) \cdot J_\alpha(x, z, w) + J_\alpha(y, z, w) \cdot \alpha^2(x), \\
J_\alpha(y \cdot z, \alpha(w), \alpha(x)) &= G(y, z, w, x) + \alpha^2(z) \cdot J_\alpha(y, w, x) + J_\alpha(z, w, x) \cdot \alpha^2(y), \\
J_\alpha(z \cdot w, \alpha(x), \alpha(y)) &= G(z, w, x, y) + \alpha^2(w) \cdot J_\alpha(z, x, y) + J_\alpha(w, x, y) \cdot \alpha^2(z).
\end{aligned}$$

Therefore, by the skew-symmetry of “ \cdot ”, $J_\alpha(x, y, z)$ and $G(w, x, y, z)$, we get

$$\begin{aligned}
&J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(x \cdot y, \alpha(z), \alpha(w)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x)) \\
&+ J_\alpha(z \cdot w, \alpha(x), \alpha(y)) \\
&= G(w, x, y, z) + G(x, y, z, w) + G(y, z, w, x) + G(z, w, x, y) \\
&= G(w, x, y, z) - G(w, x, y, z) + G(y, z, w, x) - G(y, z, w, x) \\
&= 0,
\end{aligned}$$

which proves (2.2).

Next, again from the expression of $G(w, x, y, z)$,

$$\begin{aligned}
&J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x)) \\
&= [G(w, x, y, z) + \alpha^2(x) \cdot J_\alpha(w, y, z) + J_\alpha(x, y, z) \cdot \alpha^2(w)] \\
&+ [G(y, z, w, x) + \alpha^2(z) \cdot J_\alpha(y, w, x) + J_\alpha(z, w, x) \cdot \alpha^2(y)] \\
&= 2G(w, x, y, z) - \alpha^2(w) \cdot J_\alpha(x, y, z) + \alpha^2(x) \cdot J_\alpha(y, z, w) - \alpha^2(y) \cdot J_\alpha(z, w, x) \\
&+ \alpha^2(z) \cdot J_\alpha(w, x, y)
\end{aligned}$$

so that we get (2.3). \square

From Lemma 2.5 and Lemma 2.6, we get the following expression of $G(w, x, y, z)$.

Lemma 2.7. *Let (A, \cdot, α) be a Hom-Malcev. Then*

$$G(w, x, y, z) = 2[J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x))] \quad (2.4)$$

for all w, x, y, z in A .

Proof. Set $g(w, x, y, z) = J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(x \cdot y, \alpha(z), \alpha(w)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x)) + J_\alpha(z \cdot w, \alpha(x), \alpha(y))$. Then (2.2) says that $g(w, x, y, z) = 0$ for all w, x, y, z in A . Now, by adding $g(w, x, y, z) - g(x, w, y, z)$ to the right-hand side of Lemma 2.4(ii), we get

$$\begin{aligned}
\alpha^2(w) \cdot J_\alpha(x, y, z) &- \alpha^2(x) \cdot J_\alpha(y, z, w) \\
&+ \alpha^2(y) \cdot J_\alpha(z, w, x) - \alpha^2(z) \cdot J_\alpha(w, x, y) \\
&= J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x)) \\
&+ J_\alpha(w \cdot y, \alpha(z), \alpha(x)) + J_\alpha(z \cdot x, \alpha(w), \alpha(y)) \\
&- J_\alpha(z \cdot w, \alpha(x), \alpha(y)) - J_\alpha(x \cdot y, \alpha(z), \alpha(w)) \\
&+ J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(x \cdot y, \alpha(z), \alpha(w)) \\
&+ J_\alpha(y \cdot z, \alpha(w), \alpha(x)) + J_\alpha(z \cdot w, \alpha(x), \alpha(y))
\end{aligned}$$

$$\begin{aligned}
& - J_\alpha(x \cdot w, \alpha(y), \alpha(z)) - J_\alpha(w \cdot y, \alpha(z), \alpha(x)) \\
& - J_\alpha(y \cdot z, \alpha(x), \alpha(w)) - J_\alpha(z \cdot x, \alpha(w), \alpha(y)) \\
& = 3J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + 3J_\alpha(y \cdot z, \alpha(w), \alpha(x))
\end{aligned}$$

i.e.

$$\begin{aligned}
\alpha^2(w) \cdot J_\alpha(x, y, z) & - \alpha^2(x) \cdot J_\alpha(y, z, w) \\
& + \alpha^2(y) \cdot J_\alpha(z, w, x) - \alpha^2(z) \cdot J_\alpha(w, x, y) \\
& = 3[J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x))]. \quad (2.5)
\end{aligned}$$

Next, adding (2.3) and (2.5) together, we get

$$\begin{aligned}
2G(w, x, y, z) & - \alpha^2(w) \cdot J_\alpha(x, y, z) + \alpha^2(x) \cdot J_\alpha(y, z, w) \\
& - \alpha^2(y) \cdot J_\alpha(z, w, x) + \alpha^2(z) \cdot J_\alpha(w, x, y) \\
& + \alpha^2(w) \cdot J_\alpha(x, y, z) - \alpha^2(x) \cdot J_\alpha(y, z, w) \\
& + \alpha^2(y) \cdot J_\alpha(z, w, x) - \alpha^2(z) \cdot J_\alpha(w, x, y) \\
& = J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x)) \\
& + 3[J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x))]
\end{aligned}$$

i.e.

$$2G(w, x, y, z) = 4[J_\alpha(w \cdot x, \alpha(y), \alpha(z)) + J_\alpha(y \cdot z, \alpha(w), \alpha(x))]$$

and (2.4) follows. \square

3 Proof

Relaying on the lemmas of section 2, we are now in position to prove the theorem.

Proof of the theorem. First we establish the identity (1.2) in a Hom-Malcev algebra. We may write (2.1) in an equivalent form:

$$\begin{aligned}
J_\alpha(w \cdot x, \alpha(y), \alpha(z)) & = \alpha^2(x) \cdot J_\alpha(w, y, z) + J_\alpha(x, y, z) \cdot \alpha^2(w) \\
& + G(w, x, y, z). \quad (2.6)
\end{aligned}$$

Now in (2.6), replace $G(w, x, y, z)$ with its expression from (2.4) to get

$$\begin{aligned}
-J_\alpha(w \cdot x, \alpha(y), \alpha(z)) & = \alpha^2(x) \cdot J_\alpha(w, y, z) + J_\alpha(x, y, z) \cdot \alpha^2(w) \\
& + 2J_\alpha(y \cdot z, \alpha(w), \alpha(x)),
\end{aligned}$$

which leads to (1.2).

Now, we proceed to prove the equivalence of (1.2) with (1.3) in an anti-commutative Hom-Malcev algebra. First assume (1.3). Then Lemmas 2.4, 2.5, 2.6, and 2.7 imply that (1.2) holds in any Hom-Malcev algebra.

Conversely, assume (1.2). Then, setting $w = y$ in (1.2), we get, by the skew-symmetry of $J_\alpha(x, y, z)$,

$$J_\alpha(y \cdot x, \alpha(y), \alpha(z)) = \alpha^2(y) \cdot J_\alpha(y, z, x) - 2J_\alpha(\alpha(y), \alpha(x), y \cdot z). \quad (2.7)$$

Now, the permutation of z with x in (2.7) gives

$$J_\alpha(y \cdot z, \alpha(y), \alpha(x)) = \alpha^2(y) \cdot J_\alpha(y, x, z) - 2J_\alpha(\alpha(y), \alpha(z), y \cdot x),$$

i.e.

$$2J_\alpha(y \cdot z, \alpha(y), \alpha(x)) = -2\alpha^2(y) \cdot J_\alpha(y, z, x) - 4J_\alpha(\alpha(y), \alpha(z), y \cdot x),$$

or

$$4J_\alpha(\alpha(y), \alpha(z), y \cdot x) = -2\alpha^2(y) \cdot J_\alpha(y, z, x) - 2J_\alpha(y \cdot z, \alpha(y), \alpha(x)). \quad (2.8)$$

Next, the subtraction of (2.8) from (2.7) gives (keeping in mind the skew-symmetry of $J_\alpha(x, y, z)$)

$$-3J_\alpha(\alpha(y), \alpha(z), y \cdot x) = 3\alpha^2(y) \cdot J_\alpha(y, z, x)$$

i.e.

$$J_\alpha(\alpha(y), \alpha(z), y \cdot x) = J_\alpha(y, z, x) \cdot \alpha^2(y)$$

so that we get (1.3). \square

Remark. If set $\alpha = Id$, then the identity (1.2) (resp. (1.3)) reduces to the identity (2.26) (resp. (2.4)) of [10]. The equivalence of (2.4) and (2.26) of [10] could be deduced from the works [10] and [11].

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